Minimum Trimmed Drag and Optimum c.g. Position

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An analytical expression is developed which shows that wing/tail interference drag is determined by wing downwash at downstream infinity. With the use of this expression, relations for minimum trimmed drag and optimum c.g. position are presented in explicit form. From this it follows that minimum induced drag is less for the combination of wing-plus-tail than for the wing alone. It is shown that this is true even for the case when the optimum tail load is a download rather than an upload. Furthermore, it is shown which are the factors that have a decisive effect on optimum c.g. position.

Nomenclature

AR	= aspect ratio
b, b_t	= wing span, tail span
C_D	= drag coefficient
C_L	= lift coefficient
C_m	= pitching moment coefficient
D	= drag
e	= Oswald's efficiency factor, $k = 1/(\pi e AR)$
e_{rel}	= e_{wb}/e_t
h, h_{opt}	= c.g. position in chord length, optimum c.g. position
h_{wb}, h_t	= aerodynamic center in chord length for wing-body combination, tail
k	= lift-depended drag, $C_D = C_{D_0} + kC_L{}^2$
L	= lift
S, S_t	= wing area, tail area
s, s_t	= wing half-span, tail half-span
V	= airspeed
w	= downwash
x, y, z	= coordinates
Γ	= circulation
$\epsilon, \epsilon_\infty$	= local downwash angle, downwash angle at downstream infinity
ϵ^*, ϵ_0^*	= downwash factors
ρ	= air density

Subscripts

t	= tail
wb	= wing body

Introduction

THE possibility of reducing trimmed drag by proper distribution of lift between the wing and the tail has recently received much attention because it offers potential net economic performance benefits. It is a primary reason for the interest in new design techniques such as ACT (Active Control Technology) or CCV (Control Configured Vehicles). In this type of design, the inherent stability of the aircraft is relaxed so that the c.g. can be moved further aft, where trimmed drag is reduced. This is feasible if the desired stability level can be maintained by a reliable stability augmentation system. The potential performance benefits have been investigated in a number of papers (see, e.g., Refs. 1-5). The possibility of reducing trimmed drag may also be of interest for more simple vehicles without stability augmentation such as general aviation aircraft.[5-8]

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Wing/tail interference drag represents one part of total trimmed drag. It is usually accounted for as follows[1-6,9]: the air flow direction at the tail location is tilted by the downwash angle due to wing circulation. Therefore, the tail lift vector perpendicular to the local flow direction has a component in the main flow direction in which overall drag is measured. This component added to overall drag is considered to represent wing/tail interference drag. As may be seen, the method described accounts for wing/tail interference drag on the basis of wing downwash at tail location. However, this is not valid since, as will be shown in this paper, wing/tail interference drag is determined by wing downwash at downstream infinity. This is not only important in regard to minimum trimmed drag but also in regard to the optimum c.g. position, since both quantities are decisively affected by wing downwash characteristics at downstream infinity.

Another point dealt with in this paper is the drag of the wing/tail combination as compared with the wing alone. It will be shown that, in regard to minimum induced drag for constant total lift, the combination of wing-plus-tail is superior to the wing alone. An additional purpose of this paper is to show the conditions when the minimum trimmed drag configuration requires a download on the tail rather than an upload. Here again, wing downwash at downstream infinity is a decisive factor. The optimum configuration with a download on the tail may be of particular interest since it is usually assumed that trimmed drag can be reduced by moving the c.g. further aft, causing a reduced download on the tail, or possibly even an upload. Furthermore, the parameters are identified which move the optimum c.g. position forward. This may be of particular interest for inherently stable vehicles in order to achieve as small trimmed drag as possible.

Wing/Tail Interference Drag

The equation for overall drag may be expressed as (where, for conveniece, dynamic pressure ratio is assumed to be $\bar{q}_t/\bar{q} = 1$):

$$C_D = C_{D_0} + k_{wb}C_{L_{wb}}^2 + (S_t/S)k_tC_{L_t}^2 + C_{D_{int}} \qquad (1)$$

with

$$C_{D_0} = C_{D_{0_{wb}}} + (S_t/S)C_{D_{0_t}}$$

The terms $C_{D0_{wb}}$ and C_{D0_t} represent wing-body and tail drag at zero lift. The other terms denote induced drag. From Ref. 10 it follows that the total induced drag of a multiplane system (here: wing-body plus tail) is made up of the induced drag of each lifting element treated separately plus an additional drag contribution which results from the mutual interference between the vortices of each lifting element. In the case considered here, the latter represents wing/tail interference drag denoted by $C_{D_{int}}$. It may be determined by the method developed in the following. The incremental tail drag d^2D_{t-w} resulting from the interaction of tail circulation Γ_t and the downwash dw_{t-w} induced by wing circulation may be expressed as

$$d^2D_{t-w} = \rho \Gamma_t dy_t dw_{t-w} \tag{2}$$

The downwash dw_{t-w} is given by

$$dw_{t-w} = \frac{1}{4\pi} \frac{\Gamma_w dy_w}{(y_t - y_w)^2} \left(1 - \frac{x_t - x_w}{r}\right) \tag{3}$$

where

$$r = \sqrt{(x_t - x_w)^2 + (y_t - y_w)^2} \tag{4}$$

Similar expressions exist for the incremental drag d^2D_{w-t} at the wing induced by tail circulation. Combining them with Eqs. (2) and (3) and integrating yield the following expression for total interference drag (for details, see Ref. 11):

$$D_{int} = \frac{\rho}{2\pi} \int_{-s_t}^{s_t} \Gamma_t dy_t \int_{-s_w}^{s_w} \Gamma_w \frac{dy_w}{(y_t - y_w)^2} \tag{5}$$

The second part on the right-hand side can be interpreted as wing downwash angle at downstream infinity, $\epsilon_\infty(y_t)$. It is given by

$$\epsilon_\infty(y_t) = \frac{1}{2\pi V} \int_{-s_w}^{s_w} \Gamma_w \frac{dy_w}{(y_t - y_w)^2} \tag{6}$$

Thus

$$D_{int} = \rho V \int_{-s_t}^{s_t} \epsilon_\infty \Gamma_t dy_t$$

The spanwise distribution of $\epsilon_\infty(y_t)$ does not change very much in the midsection considered, the length of which is equal to tail span (i.e., $-s_t \leqq y_t \leqq s_t$). This is because tail span is considerably smaller than wing span for usual configurations. Therefore, $\epsilon_\infty(y_t)$ may be substituted by a mean value $\bar{\epsilon}_\infty$ such that

$$D_{int} = \bar{\epsilon}_\infty \rho V \int_{-s_t}^{s_t} \Gamma_t dy_t \tag{7}$$

In this equation, the integral combined with the factors ρ V represents tail lift L_t. Thus, Eq. (7) may be rewritten as

$$D_{int} = \bar{\epsilon}_\infty L_t \tag{8a}$$

In coefficient form, it may be expressed as

$$C_{D_{int}} = \bar{\epsilon}_\infty C_{L_t} S_t / S \tag{8b}$$

The method currently used for accounting for wing/tail interference drag is based, as noted in the introduction, on local downwash at the tail. This means that it overestimates wing/tail interference drag. An example for local downwash is shown in Fig. 1. From this it follows that the overestimate may be larger than 50% when the longitudinal tail displacement is less than a half-span behind the wing. A second point to be mentioned refers to the effect of longitudinal tail position on wing tail interference drag. The current method implies a rather great effect, as may be seen in Fig. 1 from the large change of local downwash in longitudinal direction. However, Eqs. (8a) and (8b) show that wing/tail interference drag is independent of longitudinal tail position. This is valid for the usual range of tail displacements.

The relations described by Eqs. (8a) and (8b) are valid for coplanar wing/tail configurations. In Ref. 11, it is shown that they can also be applied to noncoplanar configurations, if downwash $\bar{\epsilon}_\infty$ accounts for the vertical displacement z_t of the tail, i.e. if $\bar{\epsilon}_\infty(0)$ is substituted by $\bar{\epsilon}_\infty(z_t)$.

Minimum Trimmed Drag

The overall drag expression of Eq. (1), including wing/tail interference drag according to Eq. (8b), is to be evaluated for constant total lift, $C_L = \text{const}$, and for pitching moment equilibrium, $C_m = 0$. The coefficients C_L and C_m may be expressed as

$$C_L = C_{L_{wb}} + (S_t/S) C_{L_t} \tag{9a}$$

$$C_m = C_{m_{0_{wb}}} + C_L (h - h_{wb}) - (S_t/S) C_{L_t} (h_{wb} - h_t) \tag{9b}$$

In regard to downwash, it is assumed that $\bar{\epsilon}_\infty$ is made up of two parts

$$\bar{\epsilon}_\infty = (\bar{\epsilon}_\infty)_0 + (\bar{\epsilon}_\infty)_l \tag{10}$$

Part $(\bar{\epsilon}_\infty)_0$ represents a constant independent of wing lift, and part $(\bar{\epsilon}_\infty)_l$ can be considered to be a linear function of wing lift, i.e. $(\bar{\epsilon}_\infty)_l \propto C_{L_{wb}}$. It is appropriate for the evaluation of the drag equation to define downwash factors as follows:

$$\epsilon_0^* = (\bar{\epsilon}_\infty)_0 / (2k_{wb}) \tag{11a}$$

$$\epsilon^* = (\bar{\epsilon}_\infty)_l / (2k_{wb} C_{L_{wb}}) \tag{11b}$$

Thus, the downwash term depending on wing lift $C_{L_{wb}}$ can now be treated as a constant, too.

With the use of Eqs. (9-11) and the relations $(S/S_t)k_t/k_{bw} = e_{rel}(b/b_t)^2$ and $e_{rel} = e_{wb}/e_t$, the following expression of minimum trimmed drag can be derived:

$$C_{D_{min}} = C_{D_0} + C_{D_{i_{min}}} \tag{12a}$$

where

$$C_{D_{i_{min}}} = k_{wb} C_L^2 \left(1 - \frac{(1 - \epsilon^* - \epsilon_0^*/C_L)^2}{1 + e_{rel}(b/b_t)^2 - 2\epsilon^*}\right. \tag{12b}$$

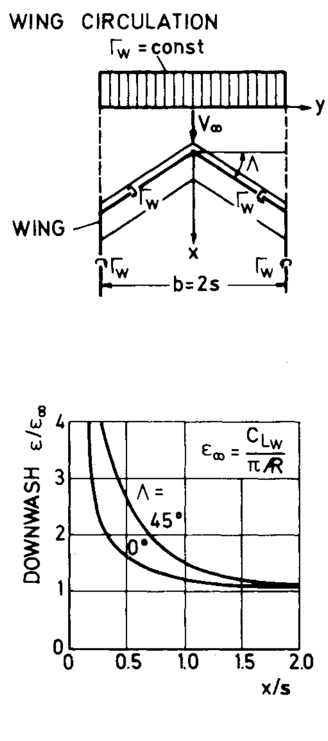

Fig. 1 Local downwash (from Ref. 12).

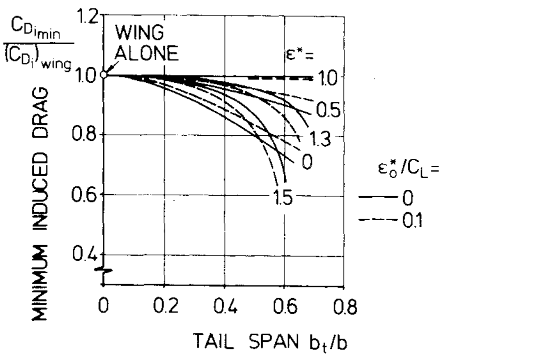

Fig. 2 Effect of tail span and downwash on minimum trimmed drag.

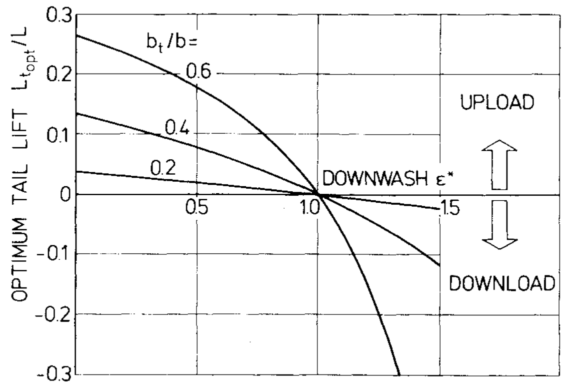

Fig. 3 Optimum tail lift ($\epsilon_0^* = 0$).

These expressions show that minimum trimmed drag is made up of zero lift drag and minimum induced drag. The bracketed term in Eq. (12b) represents tail influence on minimum induced drag. Because of the numerator and denominator being always positive, the bracketed term is less than one. From this it follows that – for a given total lift – minimum induced drag is less for the combination of wing plus tail than for the wing alone, the drag of which is represented by the term $k_{wb}\ C_L^2$ in Eq. (12b). The decisive factors are tail span and wing downwash at downstream infinity. This is illustrated in Fig. 2. Increasing tail span reduces minimum induced drag. In regard to downwash effect, it follows that minimum induced drag is reduced not only by small but also by large downwash angles. This is because the numerator in the bracketed term of Eq. (12b) is quadratic. Small downwash values ($\epsilon^* + \epsilon_0^*/C_L < 1$) may correspond to high tail configurations. Large values ($\epsilon^* + \epsilon_0^*/C_L > 1$) may correspond to coplanar wing/tail configurations where, in addition, spanwise wing lift loading may be high in the midsection. The only case where minimum induced drag of the wing/tail combination is equal to the drag of the wing alone is given by $\epsilon^* + \epsilon_0^*/C_L = 1$. This may be of particular interest since, for coplanar wing/tail combinations with $\epsilon_0^* = 0$, it represents the case where the spanwise lift distribution of the wing is elliptical. In this case, downwash is given by[12] $(\bar{\epsilon}_\infty)_I = 2C_{L_{wb}}/(\pi A\!\!\!R)$ and $k_{wb} = 1/(\pi A\!\!\!R)$ so that, from Eq. (11b), $\epsilon^* = 1$. In addition, this case represents the criterion of whether there must be an up- or a download on the tail in order to achieve the optimum lift distribution between the wing and the tail. An example is presented in Fig. 3 which shows that large downwash angles require a download on the tail. The opposite is true when downwash is small. This is also shown by the following analytical expression for optimum tail lift

$$\frac{L_{t_{\mathrm{opt}}}}{L} = \frac{1 - \epsilon^* - \epsilon_0^*/C_L}{1 + e_{\mathrm{rel}}(b/b_t)^2 - 2\epsilon^*} \tag{13}$$

A download on the tail requires, for constant total lift, the wing to have more lift. This leads to additional wing drag. Despite this fact, minimum induced drag of the wing/tail combination is decreased as compared with the wing alone (when having the same lift as the wing/tail combination). A physical explanation is presented in Fig. 4. It is shown that the tail lift vector is tilted by local downwash angle, thus having a component forward. For $\epsilon^* + \epsilon_0^*/C_L > 1$, this component is greater than the additional wing and tail drag caused by the tail download. As a result, overall drag is reduced. It is evident from Fig. 4 that this effect is larger the more the tail lift vector is tilted, i.e. it increases with downwash angle. For clarity, it may be added that the tail lift vector component acting in the main flow direction represents the tail contribution to total wing/tail interference drag.

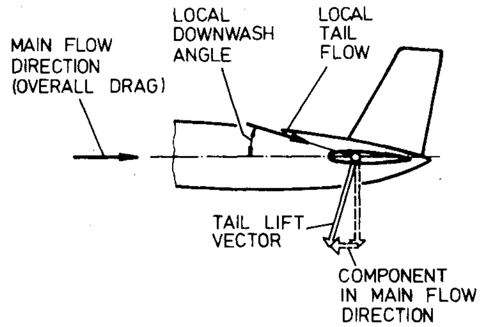

Fig. 4 Tail lift component reducing overall drag.

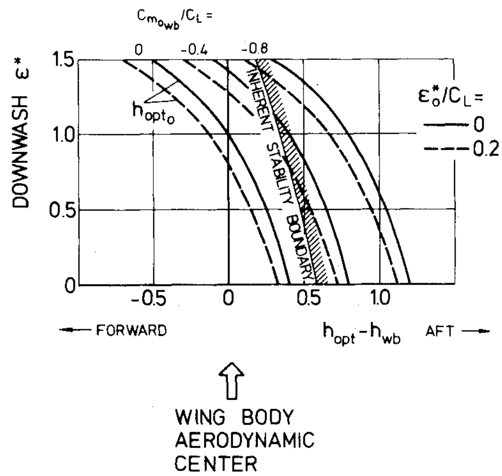

Fig. 5 Optimum c.g. position.

Optimum c.g. Position

The optimum c.g. position represents the c.g. position related to minimum trimmed drag. It may be expressed in the following form:

$$h_{\mathrm{opt}} = h_{\mathrm{opt}_0} - C_{m_{0_{wb}}}/C_L \tag{14}$$

The first term on the right-hand side represents the case when $C_{m_{0_{wb}}} = 0$. It may be written as

$$h_{\mathrm{opt}_0} = h_{wb} + \frac{1 - \epsilon^* - \epsilon_0^*/C_L}{1 + e_{\mathrm{rel}}(b/b_t)^2 - 2\epsilon^*}(h_t - h_{wb}) \tag{15}$$

Here again, downwash angle at downstream infinity is a decisive factor. This is illustrated in Fig. 5. Optimum c.g. position moves forward when downwash is large. In particular, it is forward of the wing-body aerodynamic center when $\epsilon^* + \epsilon_0^*/C_L > 1$ (with $C_{m0_{wb}} = 0$). Another significant effect is due to zero lift wing-body moment $C_{m0_{wb}}$. Optimum c.g. position moves backward when as usual, $C_{m0_{wb}}$ is negative (Fig. 5). The effect of $C_{m0_{wb}}$ increases with reduction of C_L, i.e. with increase of airspeed. This is also true for the effect of zero lift downwash ϵ_0^*. On the contrary, the effect of ϵ^*, which may be the dominant downwash term at high C_L, is independent of airspeed. The effects described show that it is possible to move the optimum c.g. position forward when negative values of $C_{m0_{wb}}$ are small and downwash is large. This may be of particular interest for inherent stable vehicles without stability augmentation systems

Conclusions

It is shown that wing/tail interference drag is not determined by wing downwash at tail location but by wing downwash at downstream infinity. This is not only important in regard to minimum trimmed drag but also in regard to the optimum c.g. position, the expressions of which are presented in explicit form. It is shown that the combination of wing-plus-tail, when optimally loaded, has less induced drag than the wing alone. This is true even for the case when the optimum tail load is a download rather than an upload. The criterion for this case which is determined by wing downwash characteristics is presented. Furthermore, it is shown that minimum trimmed drag is reduced when tail span is increased. The optimum c.g. position is determined by wing downwash at downstream infinity and by zero lift wing-body moment. It moves forward when downwash is large and/or when zero lift wing-body moment has only small negative values. In such cases, the optimum c.g. position may be at a location which would yield a suitable inherent stability margin.

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